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AN IMPROVED PROCEDURE OF HYDROTRANSPORT PARAMETERS' CALCULATION FOR FLOWS IN POLYETHYLENE PIPES AND WITH FRICTION REDUCING AGENTS

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The method of determination of solid materials hydrotransport system operating point is suggested, which is suitable for analytical studies of parameters and operating regimes of hydrotransport in the wide range of slurry concentrations in inclined and horizontal pipes fabricated of polyethylene or in the presence of friction reducing agents. Use of power-law friction formula for hydraulically-smooth pipe allowed to propose a new approximation for a discharge-head characteristic of pumps using a parabola with fractional power exponent instead of the well-known approximation by a quadratic parabola. The new approximation allows to use canonical cubic equations for the determination of hydrotransport system operating point, which have analytical solutions. These equations were obtained for the cases of mono-fractional material composed of particles of the same size and density and also for poly-fractional material composed of fine, medium and coarse fractions with different flow patterns in a pipe. The proposed method allows to increase an accuracy of the operating point calculations.

KEY WORDS: hydrotransport system operating point, pump discharge-head characteristic, polyethylene pipes

The most promising methods to reach energy consumption reduction in pressurized hydrotransport systems at mineral processing factories in the modern conditions are the replacement of steel pipes by pipes fabricated of polyethylene and the adding of friction reducing agents to slurries [Baranov et al. (2006), Semenenko (2011), Shvabauer et al. (2006)]. Both methods reduce resistance and as a result of this make slurry thickening or plant pipeline elongation possible without an installation of additional pumps. However, using of these methods at the mineral processing factories is restricted by absence of reliable calculation procedures of hydrotransport parameters and regimes. On the basis of examination of more than 20 calculation procedures of slurry flow critical velocity and hydraulic gradient during slurry flow the A. Smoldyrev's calculation procedure was modernized with the purpose of calculations reliability rising. As a result the calculation procedures were elaborated realizing both mono-fractional and poly-fractional approaches to description of transporting material behaviour and also the research of regimes of pressurized hydrotransport by polyethylene pipes as well as of hydrotransport in the presence of friction reducing agents was carried out [Nykyforova (2008), Semenenko (2011), Semenenko et al. (2013)]. At the same time the calculations in accordance with the worked out procedures became more complicated because the power law was used during method elaboration for definition of dependence of friction coefficient on Reynolds number. This restricts the using of calculation procedure as values of power exponent and of proportionality constant for the power law not always are mentioned in reference literature. The power law also made the equation for determination of system operating point appreciably nonlinear when pumps are working in the network and this in quite a number of cases doesn't allow application of previously used calculation procedures. This obstacle is partly surmountable in verifying and estimating calculations, but it presents substantial difficulties during analytical study of operating regimes of hydrotransport systems and substantiation of slurry effective velocities and concentrations.

The experience of this research allowed in this work suggestion of improved procedure of hydrotransport parameters' calculation for flows in polyethylene pipes and with friction reducing agents. This calculation procedure may be used for analytical study of parameters and operating regimes of hydrotransport without precision loss. The difficulties of determination of system operating point are got round in so doing at the expense of specially elaborated approximation of discharge-head characteristic of centrifugal pumps and this allows receipt of analytical estimation of desired solution.

In the case of hydrotransport of mono-fractional material composed of particles of similar size and density the average values of these magnitudes are used. The following formulas are proposed in this case for calculation of hydraulic gradient in inclined and horizontal pipes fabricated of different materials and in the case of hydrotransport in the presence of friction reducing agents:

$$\frac{i}{\overline{\rho} i_0} = k^{2-p} + \frac{\sigma}{k}, \qquad (1)$$

$$k = \frac{V}{V_{cr}}, \qquad i_0 = \frac{N \nu^p}{2gD^{1+p}} V_{cr}^{2-p}, \qquad \sigma = \overline{\rho}^{1,5}\left(1 + 150\frac{d}{D}\right)\cos\varphi,$$

where i – slurry hydraulic gradient, meter of water column /m; $\overline{\rho}$ – slurry relative density (material/liquid density ratio); i_0 – water hydraulic gradient during flow in critical regime, meter of water column /m; k – hydrotransportation coefficient, $1 < k < 3$; σ – coefficient, which takes into account pipeline inclination angle; p – power exponent in the dependence of friction coefficient on Reynolds number, $0 < p < 2$ [ISO/TR 10501:1993, Nykyforova (2008), Semenenko et al. (2013), Shvabauer et al. (2006)]; V – average discharge velocity of slurry, m/s; V_{cr} – hydrotransportation critical velocity, m/s; N – proportionality constant in the dependence of friction coefficient on Reynolds number [ISO/TR 10501:1993, Nykyforova (2008), Semenenko et al. (2013), Shvabauer et al. (2006)]; ν – kinematic coefficient of carrying fluid viscosity, m^2/s; g – free fall acceleration m/s^2; D – pipeline diameter, m; d – average diameter of solid phase particles, m; φ – pipeline inclination angle.

The equation for calculation of system operating point is obtained as a result of substitution of expression (1) into formula for calculation of main pipeline discharge-head characteristic and equating it with discharge-head characteristic of used pumps. If discharge-head characteristic of pumps is approximated by quadratic parabola, which doesn't involve linear term, the equation for calculation of system operating point is of the form

$$\sigma_\alpha k^3 + k^{3-p} - \sigma_0 k + \sigma = 0\,, \tag{2}$$

$$\sigma_0 = \frac{\gamma - \bar{\rho} Z}{\bar{\rho} k_z L i_0}\,, \quad \sigma'_\alpha = \frac{\alpha Q_{cr}^2}{\bar{\rho} k_z L i_0}\,,$$

where γ – fictitious pump head during zero discharge, meter of water column; Z – geodetic height difference of main pipeline beginning and end, m; k_z – coefficient, which takes into account minor losses; L – main pipeline length, m; α – coefficient of pump head reduction; Q_{cr} – pump capacity through discharge tube in critical flow regime, m^3/s.

The equation (2) is appreciably nonlinear and its solution may be found only numerically. At that nonlinearity of equation (2) is affected by the term with power exponent (3 – *p*). If discharge-head characteristic of pumps is approximated by parabola with fractional power exponent and if this parabola doesn't involve linear term, the following equation is obtained instead of equation (2):

$$k^{3-p} + \sigma_\alpha k^{q+1} - \sigma_0 k + \sigma = 0\,, \tag{3}$$

$$\sigma_\alpha = \frac{\alpha Q_{cr}^q}{\bar{\rho} k_z L i_0}\,,$$

where q – power exponent in the function for approximation of discharge-head characteristic of pumps.

The equation (3) is also appreciably nonlinear, but if

$$q = 2 - p \tag{4}$$

it may be reduced to canonical form of cubic equation

$$\left(1 + \sigma_\alpha\right) k^{3-p} - \sigma_0 k + \sigma = 0 \tag{5}$$

which may be transformed into equation with two parameters by change of variables:

$$X^{3-p} - \theta X + 1 = 0\,, \tag{6}$$

$$X = \sqrt[3-p]{\frac{1+\sigma_\alpha}{\sigma}}\,k\,, \qquad \theta = \frac{\sigma_0}{\sqrt[3-p]{1+\sigma_\alpha}\,\sigma^z}\,, \qquad z = \frac{2-p}{3-p}\,.$$

The real roots of equation (6) exist if following restriction on θ value is satisfied:

$$\theta \ge \theta_*\,, \tag{7}$$

$$\theta_* = \frac{3-p}{(2-p)^z}\,, \quad X_* = \sqrt[2-p]{\frac{\theta}{3-p}}\,,$$

where θ_* – minimum value of θ; X_* – value of X, which corresponds to minimum value of θ.

If restriction (7) is satisfied the real roots of equation (6) may be determined as

$$\frac{X}{X_*} = \begin{cases} 0{,}849\left(\dfrac{\theta_*}{\theta}\right)^{1{,}658} & X < X_*, \\ 1{,}217\left(\dfrac{\theta}{\theta_*}\right)^{0{,}147} & X > X_*. \end{cases} \tag{8}$$

When using the poly-fractional approach the transporting material is divided into fine, medium and coarse fractions with different flow patterns in a pipe. The separate fractions are composed of particles of similar size and density. The average values of these magnitudes are used for each fraction. If multicomponent solid material is composed of particles with substantially different density its clustering into fine and medium fractions must be realized taking into account not their geometrical, but hydraulic size (fall velocity) [Semenenko et al. (2014)]. In this case geometrically fine fractions must be divided into fine and hydraulically medium classes with different flow patterns in a pipe and different formulas for calculation of complementary hydraulic gradient. The following formulas are proposed in this case for calculation of hydraulic gradient in inclined and horizontal pipes fabricated of different materials and in the case of hydrotransport in the presence of friction reducing agents:

$$i = J_A k^{2-p} + J_B k^{-\left(1-\frac{p}{2}\right)} + C, \tag{9}$$

$$J_A = i_0 A, \qquad J_B = \frac{B}{\sqrt{i_0}}, \qquad A = 1 + ER_1,$$

$$B = \frac{0{,}35 w_2 E}{\sqrt{2gd_2}} R_2, \qquad C = fER_3, \qquad E = \frac{(\bar{\rho}-1)(1-SR_1)}{1+(\bar{\rho}-1)SR_1} S,$$

where S – bulk concentration of fine fraction particles, parts of unity; R_1 – mass part of fine fractions (with size less than 0,15 mm) in transporting material, parts of unity; R_2 – summarized mass part of medium fractions (with size more than 0,15 mm and less than 2 mm) and hydraulically medium fractions in transporting material, parts of unity [Semenenko et al. (2014)]; R_3 – mass part of coarse fractions (with size more than 2 mm) in transporting material, parts of unity; w_2 – constrained fall velocity of particles of medium and hydraulically medium fractions, m/s; d_2 –average diameter of particles of medium and hydraulically medium fractions, m [Semenenko et al. (2014)]; f – generalized friction coefficient of coarse fraction particles on pipe lower wall.

When using expression (9) and condition (4) equation (5) transforms into equation

$$(J_A + J_Q) k^{3\left(1-\frac{p}{2}\right)} - J_0 k^{1-\frac{p}{2}} + J_B = 0,$$

which after substitution of

$$k = \left(\frac{J_B}{J_A + J_Q} \right)^{\frac{2}{3(2-p)}} x^{\frac{2}{2-p}}$$

reduces to canonical form of cubic equation

$$x^3 - \Omega x + 1 = 0 , \tag{10}$$

$$\Omega = \frac{J_0}{\sqrt[3]{\left(J_A + J_Q\right) J_B^2}} , \qquad J_0 = \frac{\gamma - \bar{\rho} Z}{k_z L} - C , \qquad J_Q = \frac{\alpha Q_{cr}^q}{k_z L} .$$

Solution of equation (10) may be analytically obtained using Cardano method. It can be shown that the real roots of equation (10) exist only when discriminant values are negative and following condition is satisfied:

$$\Omega > \sqrt[3]{\frac{27}{4}} . \tag{11}$$

When condition (11) is satisfied the equation (10) has two real positive roots, which can be determined by following formulas:

$$k = \left(\frac{4}{3} \frac{J_0}{\left(J_A + J_Q\right) \sqrt[3]{J_B}} \right)^{\frac{1}{2-p}} \mu'^{\frac{2}{2-p}} (\Omega), \tag{12}$$

$$\mu'(\Omega) = \begin{cases} \frac{\sqrt{3}}{2} \sin \frac{\beta}{3} - \frac{1}{2} \cos \frac{\beta}{3} \\ \cos \frac{\beta}{3} \end{cases} , \qquad \beta = \arccos\left(- \sqrt{\frac{27}{4\Omega^3}} \right) .$$

If transporting material doesn't contain particles of medium and hydraulically medium fractions the following expression must be used instead of formula (12):

$$k = \sqrt[2-p]{\frac{J_0}{J_A + J_Q}} . \tag{13}$$

So it is obvious from formulas (8), (12) and (13) that suggested method of approximation of discharge-head characteristic of pumps with regard for power exponent in the dependence of friction coefficient on Reynolds number namely

$$H = \gamma - \alpha Q^{2-p} \tag{14}$$

allows obtaining of analytical solution of nonlinear equations. Previously when determining of system operating point nonlinear equations were solved only numerically. For accuracy evaluation of suggested method the data of discharge-head characteristic of the pump GIW WBC 54 (HD) with runner rotational speed 550 rev. /min. [Baranov et al. (2006)] were used. The approximation of discharge-head characteristic of this pump for different values of power exponent p was carried out using formula (14) and least squares method. The results of this approximation are presented in table 1.

Table 1
The results of approximation of discharge-head characteristic of the pump GIW WBC 54 (HD) with runner rotational speed 550 rev./min.

Discharge, m^3/s	Rated head, m w.c.	Calculated head, meter of water column								
		Values of power exponent *p*								
		0	0,17	0,26	0,27	0,29	0,32	0,4	0,55	0,6
0,000	107,2	107,2	105,1	105,5	105,6	105,7	105,8	106,3	107,2	107,5
0,139	106,4	106,4	104,8	105,2	105,3	105,3	105,5	105,8	106,5	106,7
0,278	104,8	104,8	104,2	104,5	104,5	104,6	104,7	104,9	105,3	105,4
0,417	103,2	103,2	103,3	103,4	103,4	103,4	103,5	103,6	103,7	103,8
0,556	101,6	101,9	102,0	102,0	102,0	102,0	102,0	102,0	101,9	101,9
0,694	100,8	100,5	100,4	100,4	100,3	100,3	100,3	100,2	99,9	99,8
0,833	97,6	98,8	98,6	98,4	98,4	98,4	98,3	98,1	97,7	97,6
0,972	95,2	96,9	96,5	96,2	96,2	96,2	96,1	95,8	95,3	95,2
1,111	92,8	94,6	94,1	93,8	93,8	93,7	93,6	93,3	92,8	92,6
1,250	90,4	92,0	91,5	91,2	91,1	91,0	90,9	90,7	90,1	89,9
1,389	88,0	89,1	88,6	88,3	88,2	88,2	88,1	87,8	87,3	87,1
1,528	84,8	85,9	85,4	85,1	85,1	85,1	85,0	84,8	84,4	84,2
1,667	81,6	82,4	82,0	81,8	81,8	81,8	81,7	81,5	81,3	81,2
1,806	78,4	78,6	78,4	78,3	78,3	78,2	78,2	78,2	78,1	78,1
1,944	74,4	74,5	74,5	74,5	74,5	74,5	74,6	74,6	74,8	74,9
2,083	71,2	70,0	70,4	70,6	70,6	70,6	70,7	70,9	71,4	71,5
2,222	67,2	65,3	66,0	66,4	66,4	66,5	66,7	67,1	67,9	68,1

The analysis of calculation results (table 1) shows that maximum relative difference of head for different values of power exponent *p* doesn't exceed 2 %. Such accuracy is quite satisfactory for hydraulic calculation of pipeline transport systems at mineral processing factories. It should be noted that traditional approximation of discharge-head characteristic of pumps by quadratic parabola gives error of 3 %.

The analysis of values of approximation coefficients in formula (14) depending on values of power exponent *p* shows that fictitious pump head during zero discharge γ weakly depends on *p,* but coefficient of pump head reduction α strongly depends on *p*. It is convenient for considered calculations to present the dependences of these magnitudes on *p* in the following form:

$$\gamma_p = G\gamma_0, \qquad \alpha_p = U\alpha_0,$$

$$G = 1 + 0{,}0381p + 0{,}0215p^2, \qquad U = 1 + 0{,}7781p + 0{,}4477p^2,$$

where γ_p – fictitious pump head during zero discharge for arbitrary value of p; G – adjustment coefficient for fictitious pump head during zero discharge; γ_0 – fictitious pump head during zero discharge when $p = 0$; α_p – coefficient of pump head reduction for arbitrary value of p; U – adjustment coefficient for α_p; α_0 – coefficient of pump head reduction when $p = 0$.

The dependence of adjustment coefficients values G and U on parameter p is shown on the fig. 1.

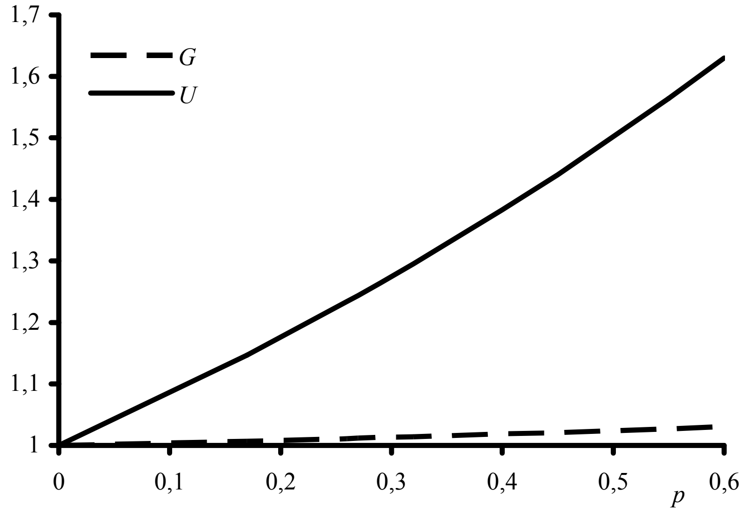

Fig. 1 Dependence of adjustment coefficients values G and U on parameter p

CONCLUSIONS

1. The suggested approximation of discharge-head characteristic of pumps by a parabola using the exponent from the power-law friction formula for a hydraulically-smooth pipe allowed a derivation of canonical cubic equations for a determination of the hydrotransport system operating point, which have analytical solutions.
2. Used mono-fractional and poly-fractional approaches cover all cases of solid particles hydrotransport.
3. The suggested method of the determination of the system operating point is suitable for analytical studies of hydrotransport parameters and operating regimes in the wide range of slurry concentrations in inclined and horizontal pipes fabricated of polyethylene or in the presence of friction reducing agents.
4. The suggested calculation procedure may increase an accuracy of the calculation of the system operating point as well as a reduction of hydrotransport energy consumption.
5. As long as the power-law friction formula is valid also for pipeline systems transporting water or other homogeneous liquids, the suggested calculation procedure may be used not only for hydrotransport systems of mineral processing factories.

REFERENCES

1. Баранов, Ю.Д., Блюсс, Б.А., Семененко, Е.В., Шурыгин, В.Д., 2006. Обоснование параметров и режимов работы систем гидротранспорта горных предприятий (Baranov, Yu.D., Blyuss, B.A., Semenenko, Eu.V., Shurygin, V.D., 2006. Substantiation of Parameters and Operating Regimes of Hydrotransport Systems of Mineral Processing Factories), Новая идеология, Днепропетровск.
2. ISO/TR 10501:1993. Thermoplastics pipes for the transport of liquids under pressure. Calculation of head losses, Estonian Centre for Standardisation, Tallin.
3. Никифорова Н.А., 2008. Обоснование параметров процесса гидротранспортирования минерального сырья с использованием гидродинамически активных веществ (Nykyforova N.A., 2008. Substantiation of parameters of hydrotransportation process of minerals with using of friction reducing agents), Dnipropetrovsk. Unpublished PhD diss. Institute of Geotechnical Mechanics of the National Academy of Sciences of Ukraine.
4. Семененко Е.В., 2011. Научные основы технологий гидромеханизации открытой разработки титан-цирконовых россыпей (Semenenko, Eu.V., 2011. The Scientific Foundation of Technologies of Hydromechanization of Quarry Operation of titanium-zircon placers), Наукова думка, Киев.
5. Семененко Е.В., Никифорова Н.А., Татарко Л.Г., 2013. Расчет режимов работы гидротранспортных комплексов с магистралями, состоящими из участков полиэтиленовых и стальных труб (Semenenko, Eu.V., Nykyforova N.A., Tatarko, L.G, 2013. Calculation of operating modes of hydrotransport complexes with main pipelines composed of polyethylene and steel pipes sections. Collected articles Geotechnical Mechanics 111, 140-152), Межведомственный сборник научных трудов Геотехническая механика 111, 140-152.
6. Semenenko, Eu., Nykyforova N., Tatarko, L., 2014. The method of hydraulic gradient and critical velocity calculation for hydrotransportation of particles with substantially different densities. Proc. 15th Int. Freight Pipeline Society Symposium, Czech Republic (Prague), 24-27 June, 2014, 248-256.
7. Швабауэр, В., Гвоздев И., Гориловский М., 2006. Расчет гидравлических потерь давления в трубопроводе из пластмасс, (Shvabauer, V., Gvozdyov, I., Gorilovskiy, M., 2006. Calculation of hydraulic pressure loss in plastic pipeline. J. Polymeric Pipes. Ukraine 1, 46-52), Журнал Полимерные трубы. Украина 1, 46-52.